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# NAVAL AIR DEVELOPMENT CENTER

Warminster, Pennsylvania

SYSTEMS ANALYSIS AND ENGINEERING DEPARTMENT

TECHNICAL MEMORANDUM 70-014-3

1 MARCH 1971

## A MODEL FOR OBTAINING AN ESTIMATE
## FOR PROBABILITY OF ENTRAPMENT BY AN ENEMY RADAR
## IN THE PRESENCE OF NOISE JAMMING

AIRTASK A5335336/202C/0W3344000
Work Unit 1

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Prepared by: _Irwin R. Goodman_
Irwin R. Goodman

Reviewed by: _R. N. Becker_
R. N. Becker

Approved by: _D. F. MacInnes_
D. F. MacInnes
Chief Systems Analyst

# INTRODUCTION

The model presented in this technical memorandum was developed in support of the expendable jammer study currently being conducted at the Naval Air Development Center.

The purpose of this model is to evaluate the relative effectiveness of friendly jamming of enemy acquisition radars in terms of the tracking errors introduced into the enemy acquisition radar prior to target hand over to the enemy fire control radar operators. The effectiveness of friendly jamming against enemy acquisition radars was evaluated and is reported on in reference (a).

Typical results from the Radar Entrapment Model developed herein are described in reference (a). The radar entrapment model is presently programmed on the NAVAIRDEVCEN (Naval Air Development Center) CDC-3200 computer, and is useful for the evaluation of jamming performance in terms of the errors introduced into the enemy acquisition radar in terms of penetrator track prediction.

At present, the model is being expanded to include the effects of expendable jammers, that is, multiple jamming strobes, expendable jammer ballistic trajectory effects, and the effects of possible enemy radar burnthrough.

The basic radar entrapment model presented herein assumes that the enemy acquisition is tracking a target in error and is estimating the actual penetration. Track information is handed over to the enemy fire control radar which is required to position itself (in space) so as to effect an intercept with the SAM (Surface-to-Air Missile) envelope. However, due to track errors introduced by the jamming, the radar position vector is most likely in error. The model evaluates this error in terms of the probability that the aircraft will be physically present in the fire control radar entrapment region when the fire control radar is activated.

# DISCUSSION

## Description

We assume a friendly airplane (or a cluster of airplanes) carrying on board jamming devices - describes three dimensional straight line constant velocity motion.

Two enemy radar sources are present. One source, the acquisition radar, tracks the airplane until the enemy estimates, based on observed data, of when and where the airplane will most likely be first within the lethal detection region about the other source, the fire control radar. In order to seize the earliest opportunity to fire at the airplane, the latter, close to which an enemy missile launcher is located, then pivots angle-wise about its fixed location so that one corner of its lethal detection region is placed at the estimated position of entrance of the airplane within the effective missile range.

The lethal detection region for the fire control radar is defined as that region bounded by the maximal and minimal effective missile ranges and also within the finite detection cone of the radar, the position of which is angular dependent only. (See figure 1.) Estimates are also made by the enemy of the estimated exit time of the airplane, and time gates are established about the estimated entrance and exit times from the lethal region. These calculations determine the turn-on and turn-off times of the fire control radar. Since the longer the fire control radar is turned on, the more likely the enemy will itself be detected and fired upon by the aircraft, an upper bound is also imposed on the elapsed time between turn-on and turn-off times.

Standard regression analysis is used in obtaining the enemy's estimated path of the target. Outcomes are then generated at various sample times and a determination is made whether the airplane will actually be in the lethal region between enemy turn-on and turn-off times, for a given run (set of sampling outcomes). The estimate of probability of (lethal) detection is then just the percentage of runs in which the airplane passes through the lethal region between turn-on and turn-off times.

## Model Development

Parameters of equations of motion of airplane T:

$$v_x = \text{x-velocity}$$
$$v_y = \text{y-velocity}$$
$$v_z = \text{z-velocity}$$
$$x_0 = \text{initial x-position}$$
$$y_0 = \text{initial y-position}$$
$$z_0 = \text{initial z-position}$$
$$t = \text{time}; \ t_0 \text{ is initial time}$$

$$\frac{x-x_0}{v_x} = \frac{y-y_0}{v_y} = \frac{z-z_0}{v_z} = t-t_0$$

Through jamming noise, observations

$$\begin{pmatrix} \overset{\circ}{r}_i \\ \overset{\circ}{\theta}_i \\ \overset{\circ}{\phi}_i \end{pmatrix}$$

at time ti with covariance matrix

$$Cov \begin{pmatrix} \overset{\circ}{r}_i \\ \overset{\circ}{\theta}_i \\ \overset{\circ}{\phi}_i \end{pmatrix} \equiv \begin{pmatrix} \sigma_{r_i}^2 & & 0 \\ & \sigma_{\theta_i}^2 & \\ 0 & & \sigma_{\phi_i}^2 \end{pmatrix} \qquad , \; i = 1,\ldots,m \, ,$$

where for i = 1,...m (observations are made by another radar located at another source):

$\overset{\circ}{r}_i$ is the observed x-y range of T by the acquisition radar at time ti

$\overset{\circ}{\theta}_i$ is the observed azimuth angle of T by the acquisition radar at time ti

$\overset{\circ}{\phi}_i$ is the observed elevation angle of T by the acquisition radar at time ti

( See appendix A for further evaluations. )

$v_x$, $v_y$, $v_z$, $x_o$, $y_o$, $z_o$ are totally unknown to radar

The relations between x-y-z rectangular coordinates and r-θ-φ coordinates are

$$x = r \cos \theta \qquad\qquad r^2 = x^2 + y^2$$

$$y = r \sin \theta \qquad\qquad \tan \theta = \frac{y}{x}$$

$$z = r \tan \phi \qquad\qquad \tan \phi = \frac{z}{r}$$

Then using standard transformation techniques,

letting
$$\overset{\circ}{x}_i = \overset{\circ}{r}_i \cos \overset{\circ}{\theta}_i$$

$$\overset{\circ}{y}_i = \overset{\circ}{r}_i \sin \overset{\circ}{\theta}_i$$

$$\overset{\circ}{z}_i = \overset{\circ}{r}_i \tan \overset{\circ}{\phi}_i$$

$$\text{Cov}\left(\begin{pmatrix} \overset{\circ}{X}_i \\ \overset{\circ}{Y}_i \\ \overset{\circ}{Z}_i \end{pmatrix}\right) \equiv \begin{pmatrix} \sigma^2_{x_i} & \sigma_{x_i y_i} & \sigma_{x_i z_i} \\ \sigma_{x_i y_i} & \sigma^2_{y_i} & \sigma_{y_i z_i} \\ \sigma_{x_i z_i} & \sigma_{y_i z_i} & \sigma^2_{z_i} \end{pmatrix} \approx J \cdot \text{Cov}\left(\begin{pmatrix} \overset{\circ}{n} \\ \overset{\circ}{\theta}_i \\ \overset{\circ}{\phi}_i \end{pmatrix}\right) \cdot J^T$$

where

$$J = \left(\dfrac{d\begin{pmatrix} x \\ y \\ z \end{pmatrix}}{d\begin{pmatrix} r \\ \theta \\ \phi \end{pmatrix}}\right)_{\substack{at \\ }} \quad \substack{r = r_i \\ \theta = \theta_i \\ \phi = \phi_i}$$

$$= \begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\phi & r\cos\theta & 0 \\ \tan\phi & 0 & r\sec^2\phi \end{pmatrix}_{at} \quad \substack{r = r_i \\ \theta = \theta_i \\ \phi = \phi_i}$$

Each $\begin{pmatrix} \overset{\circ}{r}_i \\ \overset{\circ}{\theta}_i \\ \overset{\circ}{\phi}_i \end{pmatrix}$ is normally distributed and for $i \neq j$, $\begin{pmatrix} \overset{\circ}{r}_i \\ \overset{\circ}{\theta}_i \\ \overset{\circ}{\phi}_i \end{pmatrix}$ is statistically

independent of $\begin{pmatrix} \overset{\circ}{r}_j \\ \overset{\circ}{\theta}_j \\ \overset{\circ}{\phi}_j \end{pmatrix}$, $1 \leq i, j \leq m$.

$(\vphantom{)}$

Letting $x_i$, $y_i$, $z_i$ be the actual x-y-z coordinate values of T at time $t_i$ (unknown to acquisition radar) and $r_i$, $\theta_i$, $\phi_i$ the corresponding r-$\theta$-$\phi$ coordinates, then since

$$x_i = x_0 + v_x \cdot (t_i - t_0)$$

$$y_i = y_0 + v_y \cdot (t_i - t_0)$$

$$z_i = z_0 + v_z \cdot (t_i - t_0)$$

we can compute

whe

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$$r_i^2 = x_i^2 + y_i^2$$

$$\tan \theta_i = \frac{y_i}{x_i} \ , \ \text{etc.}$$

$$i = 1,\ldots,m$$

$$\tan \phi_i = \frac{z_i}{r_i} \ , \ \text{etc.}$$

Then by use of standard tables for each run, we generate the outcomes $\overset{\circ}{r}_i$ from $N(r_i,\sigma_{r_i}^2)$, $\overset{\circ}{\theta}_i$ from $N(\theta_i, \sigma_{\theta_i}^2)$ and $\overset{\circ}{\phi}_i$ from $N(\phi_i, \sigma_{\phi_i}^2)$. In turn, from the relations developed previously, we then obtain $\overset{\circ}{x}_i, \overset{\circ}{y}_i, \overset{\circ}{z}_i$.

Then, we have for each run the following regression model:

$$
\underbrace{\begin{pmatrix} \overset{\circ}{x}_1 \\ \overset{\circ}{y}_1 \\ \overset{\circ}{z}_1 \\ \vdots \\ \overset{\circ}{x}_m \\ \overset{\circ}{y}_m \\ \overset{\circ}{z}_m \end{pmatrix}}_{\substack{Y_m \\ (3m \text{ by } 1)}}
=
\underbrace{\begin{pmatrix}
1 & (t_1-t_0) & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & (t_1-t_0) & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & (t_1-t_0) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & (t_m-t_0) & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & (t_m-t_0) & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & (t_m-t_0)
\end{pmatrix}}_{\substack{B_m \\ (3m \text{ by } 6 \text{ of rank } 6)}}
\cdot
\underbrace{\begin{pmatrix} X_0 \\ V_x \\ Y_0 \\ V_y \\ Z_0 \\ V_z \end{pmatrix}}_{\substack{X \\ (6 \text{ by } 1)}}
+
\underbrace{\begin{pmatrix} \varepsilon_{x_1} \\ \varepsilon_{y_1} \\ \varepsilon_{z_1} \\ \vdots \\ \varepsilon_{x_m} \\ \varepsilon_{y_m} \\ \varepsilon_{z_m} \end{pmatrix}}_{\substack{\varepsilon_m \\ (3m \text{ by } 1)}}
$$

$$\varepsilon_m \text{ is } N(0_{3m,1} , \Sigma_m)$$

$$
\text{where } \Sigma_m = \begin{pmatrix}
\sigma_{x_1}^2 & \sigma_{x_1 y_1} & \sigma_{x_1 z_1} & & & \\
\sigma_{x_1 y_1} & \sigma_{y_1}^2 & \sigma_{y_1 z_1} & & & \\
\sigma_{x_1 z_1} & \sigma_{y_1 z_1} & \sigma_{z_1}^2 & & & \\
& & & \ddots & & \\
& & & & \sigma_{x_m}^2 & \sigma_{x_m y_m} & \sigma_{x_m z_m} \\
& & & & \sigma_{x_m y_m} & \sigma_{y_m}^2 & \sigma_{y_m z_m} \\
& & & & \sigma_{x_m z_m} & \sigma_{y_m z_m} & \sigma_{z_m}^2
\end{pmatrix}
$$

is $3m$ by $3m$ positive definite

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It follows from standard results (reference (a)) that "best" estimate of X

    = minimal covariance matrix absolute unbiased estimate

    = generalized least squares estimate, etc.
based on data $\overset{\circ}{x}_i$, $\overset{\circ}{y}_i$, $\overset{\circ}{z}_i$, $t_i$, $\quad i = 1,\ldots m$

    $= \hat{X}_m$

$$= \begin{pmatrix} \hat{X}_{m0} \\ \hat{V}_{mx} \\ \hat{Y}_{m0} \\ \hat{V}_{mY} \\ \hat{Z}_{m0} \\ \hat{V}_{mz} \end{pmatrix}$$

$$= \left( B_m^T \, \Sigma_m^{-1} \, B_m \right)^{-1} B_m^T \, \Sigma_m^{-1} \, Y_m$$

by matrix partitioning, letting $Y_{(i)} \overset{df}{=} \begin{pmatrix} \overset{\circ}{x}_i \\ \overset{\circ}{y}_i \\ \overset{\circ}{z}_i \end{pmatrix}$

$B_{(i)} \overset{df}{=} \begin{pmatrix} 1 & (t_i-t_0) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & (t_i-t_0) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & (t_i-t_0) \end{pmatrix}$, $\Sigma_{(i)} \overset{df}{=} \begin{pmatrix} \sigma_{x_i}^2 & \sigma_{x_i y_i} & \sigma_{x_i z_i} \\ \sigma_{x_i y_i} & \sigma_{y_i}^2 & \sigma_{y_i z_i} \\ \sigma_{x_i z_i} & \sigma_{y_i z_i} & \sigma_{z_i}^2 \end{pmatrix}$;

$$= \underbrace{\left( \sum_{i=1}^{m} B_{(i)}^T \, \Sigma_{(i)}^{-1} \, B_{(i)} \right)^{-1}}_{6 \text{ by } 6 \text{ matrix}} \cdot \sum_{i=1}^{m} B_{(i)}^T \, \Sigma_{(i)}^{-1} \, Y_{(i)}$$

We then have
$$\hat{X}_m \equiv \hat{X}_m = \hat{X}_{m0} + \hat{V}_{mx} \cdot (t_m - t_0)$$
$$\hat{Y}_m \equiv \hat{Y}_m = \hat{Y}_{m0} + \hat{V}_{mY} \cdot (t_m - t_0)$$
$$\hat{Z}_m \equiv \hat{Z}_m = \hat{Z}_{m0} + \hat{V}_{mz} \cdot (t_m - t_0)$$

The above results constitute the "direct" regression solution. However, a 6 by 6 matrix must be inverted for each new observation time, $t_{m+1}$. Recursive filtering as given by the Kalman-Buoy filter reduces this problem:

Let

$$
A(t_q - t_p) \stackrel{df}{=}
\begin{pmatrix}
\begin{array}{cc} 1 & (t_q-t_p) \\ 0 & 1 \end{array} & \bigcirc & \bigcirc \\
\bigcirc & \begin{array}{cc} 1 & (t_q-t_p) \\ 0 & 1 \end{array} & \bigcirc \\
\bigcirc & \bigcirc & \begin{array}{cc} 1 & (t_q-t_p) \\ 0 & 1 \end{array}
\end{pmatrix} \quad ,
$$

$$
M_o \stackrel{df}{=}
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix} \quad , \quad
X_p \stackrel{df}{=}
\begin{pmatrix}
X_p \\ V_x \\ Y_p \\ V_y \\ Z_p \\ V_z
\end{pmatrix} \quad .
$$

It is easily seen that

$$
X_q = A(t_q - t_p) \cdot X_p
$$

Then if we let $\hat{X}_p[p+1]$ be the "best" predicted value of $X_{p+1}$ based on data up to time $t_p$, we can show (see reference (a) for example)

(i) $\hat{X}_{p+1} = \hat{X}_{p}{}_{p+1} + Cov(\hat{X}_{p}{}_{p+1}) \cdot M_o^T \cdot (\Sigma_{(p+1)} + M_o \cdot Cov(\hat{X}_{p}{}_{p+1}) \cdot M_o^T)^{-1} \cdot$

$$\cdot (\underline{Y}_{(p+1)} - M_o \cdot \hat{X}_{p}{}_{p+1})$$

(ii) $\hat{X}_{p}{}_{p+1} = A(t_{p+1} - t_p) \cdot \hat{X}_p$

(iii) $Cov(\hat{X}_{p}{}_{p+1}) = A(t_{p+1} - t_p) \cdot Cov(\hat{X}_p) \cdot (A(t_{p+1} - t_p))^T$

(iv) $Cov(\hat{X}_p) = Cov(\hat{X}_{p-1}{}_{(p)}) - Cov(\hat{X}_{p-1}{}_{p}) \cdot M_o^T \cdot (\Sigma_{(p)} + M_o Cov(\hat{X}_{p-1}{}_{p}) \cdot M_o^T)^{-1} \cdot$

$$\cdot M_o \cdot Cov(\hat{X}_{p-1}{}_{p})$$

with initial set up:

$$(V) \quad \hat{X}_2 = \begin{pmatrix} \hat{X}_2 \\ \hat{\sigma}_{2X} \\ \hat{Y}_2 \\ \hat{\sigma}_{2Y} \\ \hat{Z}_2 \\ \hat{\sigma}_{2Z} \end{pmatrix} \equiv \begin{pmatrix} \overset{o}{X}_2 \\ (\overset{o}{X}_2 - \overset{o}{X}_1)/(t_2 - t_1) \\ \overset{o}{Y}_2 \\ (\overset{o}{Y}_2 - \overset{o}{Y}_1)/(t_2 - t_1) \\ \overset{o}{Z}_2 \\ (\overset{o}{Z}_2 - \overset{o}{Z}_1)/(t_2 - t_1) \end{pmatrix}$$

$$Cov(\hat{X}_2) = \begin{pmatrix}
\sigma_{X_2}^2 & \dfrac{\sigma_{X_2}^2}{t_2 - t_1} & \sigma_{X_2 Y_2} & \dfrac{\sigma_{X_2 Y_2}}{t_2 - t_1} & \sigma_{X_2 Z_2} & \dfrac{\sigma_{X_2 Z_2}}{t_2 - t_1} \\[2ex]
\dfrac{\sigma_{X_2}^2}{t_2 - t_1} & \dfrac{\sigma_{X_1}^2 + \sigma_{X_2}^2}{(t_2 - t_1)^2} & \dfrac{\sigma_{X_2 Y_2}}{t_2 - t_1} & \dfrac{\sigma_{X_1 Y_1} + \sigma_{X_2 Y_2}}{(t_2 - t_1)^2} & \dfrac{\sigma_{X_2 Z_2}}{t_2 - t_1} & \dfrac{\sigma_{X_1 Z_1} + \sigma_{X_2 Z_2}}{(t_2 - t_1)^2} \\[2ex]
\sigma_{X_2 Y_2} & \dfrac{\sigma_{X_2 Y_2}}{t_2 - t_1} & \sigma_{Y_2}^2 & \dfrac{\sigma_{Y_2}^2}{t_2 - t_1} & \sigma_{Y_2 Z_2} & \dfrac{\sigma_{Y_2 Z_2}}{t_2 - t_1} \\[2ex]
\dfrac{\sigma_{X_2 Y_2}}{t_2 - t_1} & \dfrac{\sigma_{X_1 Y_1} + \sigma_{X_2 Y_2}}{(t_2 - t_1)^2} & \dfrac{\sigma_{Y_2}^2}{t_2 - t_1} & \dfrac{\sigma_{Y_1}^2 + \sigma_{Y_2}^2}{(t_2 - t_1)^2} & \dfrac{\sigma_{Y_2 Z_2}}{t_2 - t_1} & \dfrac{\sigma_{Y_1 Z_1} + \sigma_{Y_2 Z_2}}{(t_2 - t_1)^2} \\[2ex]
\sigma_{X_2 Z_2} & \dfrac{\sigma_{X_2 Z_2}}{t_2 - t_1} & \sigma_{Y_2 Z_2} & \dfrac{\sigma_{Y_2 Z_3}}{t_2 - t_1} & \sigma_{Z_2}^2 & \dfrac{\sigma_{Z_2}^2}{t_2 - t_1} \\[2ex]
\dfrac{\sigma_{X_2 Z_2}}{t_2 - t_1} & \dfrac{\sigma_{X_1 Z_1} + \sigma_{X_2 Z_2}}{(t_2 - t_1)^2} & \dfrac{\sigma_{Y_2 Z_2}}{t_2 - t_1} & \dfrac{\sigma_{Y_1 Z_1} + \sigma_{Y_2 Z_2}}{(t_2 - t_1)^2} & \dfrac{\sigma_{Z_2}^2}{t_2 - t_1} & \dfrac{\sigma_{Z_1}^2 + \sigma_{Z_2}^2}{(t_2 - t_1)^2}
\end{pmatrix}$$

Here only 3 by 3 matrices need be inverted, since the only inversion required in equations (i)-(iv) is of

$$\left( \Sigma_{(p+1)} + M_0 \cdot Cov(\hat{X}_{p \ p+1}) \cdot M_0^T \right)$$

Next, at the end of m observations for a given run, we obtain $\hat{X}_m$ which gives the entire estimated equations of motion:

$$\begin{cases} \hat{X}_m(t) = \hat{X}_m + \hat{V}_{mx} \cdot (t - t_m) \\ \hat{Y}_m(t) = \hat{Y}_m + \hat{V}_{mY} \cdot (t - t_m) \\ \hat{Z}_m(t) = \hat{Z}_m + \hat{V}_{mz} \cdot (t - t_m) \end{cases}$$

If we let $\rho_o$ be the maximal effective missile range, then we consider the intersection of

$$\begin{cases} \text{Sphere of radius } \rho_o \text{ centered at origin} \\ \qquad\qquad and \\ \text{Equations of motion} \quad \begin{pmatrix} \hat{X}_m(t) \\ \hat{Y}_m(t) \\ \hat{Z}_m(t) \end{pmatrix} \end{cases}$$

(This intersection may be vacuous.)

Thus we must solve

$$\begin{cases} (\hat{X}_m + \hat{V}_{mx} \cdot (t - t_m))^2 + (\hat{Y}_m + \hat{V}_{mY} \cdot (t - t_m))^2 + \\ \qquad\qquad (\hat{Z}_m + \hat{V}_{mz} \cdot (t - t_m))^2 \qquad = \rho_o^2 \end{cases}$$

for $t = t^*_{(m)}$

Thus at time $t^*_{(m)}$ and at position $\begin{pmatrix} \hat{X}_m(t^*_{(m)}) \\ \hat{Y}_m(t^*_{(m)}) \\ \hat{Z}_m(t^*_{(m)}) \end{pmatrix} \overset{df}{=} \hat{X}_{(m)}$

the estimated motion of T will first enter the maximal missile range.

The fire control radar doctrine is then to pivot by (random) angles $\Psi$ and $\eta$, based on the data, about its source 0 (see figure 1).

Next, for the same run, estimated entrance and exit times and positions of T are computed

$$\hat{X}_{(m)} \text{ at estimated time } \hat{t}_{(m)}$$

$$\hat{X}'_{(m)} \text{ at estimated time } \hat{t}'_{(m)}$$

$\Delta_1, \Delta_2 > 0$ are predetermined constants.

Then the fire control radar is turned on at time $\hat{t}^*_{(m)} - \Delta_1$ and is turned off at min $(\hat{t}'^*_{(m)} + \Delta_1, \ \hat{t}^*_{(m)} + \Delta_2)$

Next, we consider the 0-1 random variable Z with parameter $\omega$

where

Z = 1 iff the (deterministic) target T in
following its actual path is in
lethal region L between on-times
and off-times

= 0 otherwise .

$$P_r(Z=1) = \omega$$

Thus, for a given run we assign an outcome value of Z accordingly and best estimate $\omega$ by the percentage of successful trials ; that is, $\hat{\omega} =$ number of 1's over a period of runs.

Finally, we let, after a period of runs (for the same generated actual path of T and fixed set of covariance matrices of errors) :
Probability of detection of T by radar $\approx \hat{\omega}$

To is Inner Missile Range

$\rho_0$ is Outer Missile Range

Lethal Region Against T is L

Estimated Path of T Based on Data up to Time tm

Bounding Planes for Radar Beam

Eq. of I: $(\cos\psi)X + (\sin\psi)Y - (\cot\eta)Z = 0$

Eq. of II: $(\cos\psi)X + (\sin\psi)Y - \cot(\eta+w)Z = 0$

Eq. of III: $X - (\cot\psi)Y = 0$

Eq. of IV: $X - \cot(\psi+h)Y = 0$

Parameters to be Computed:
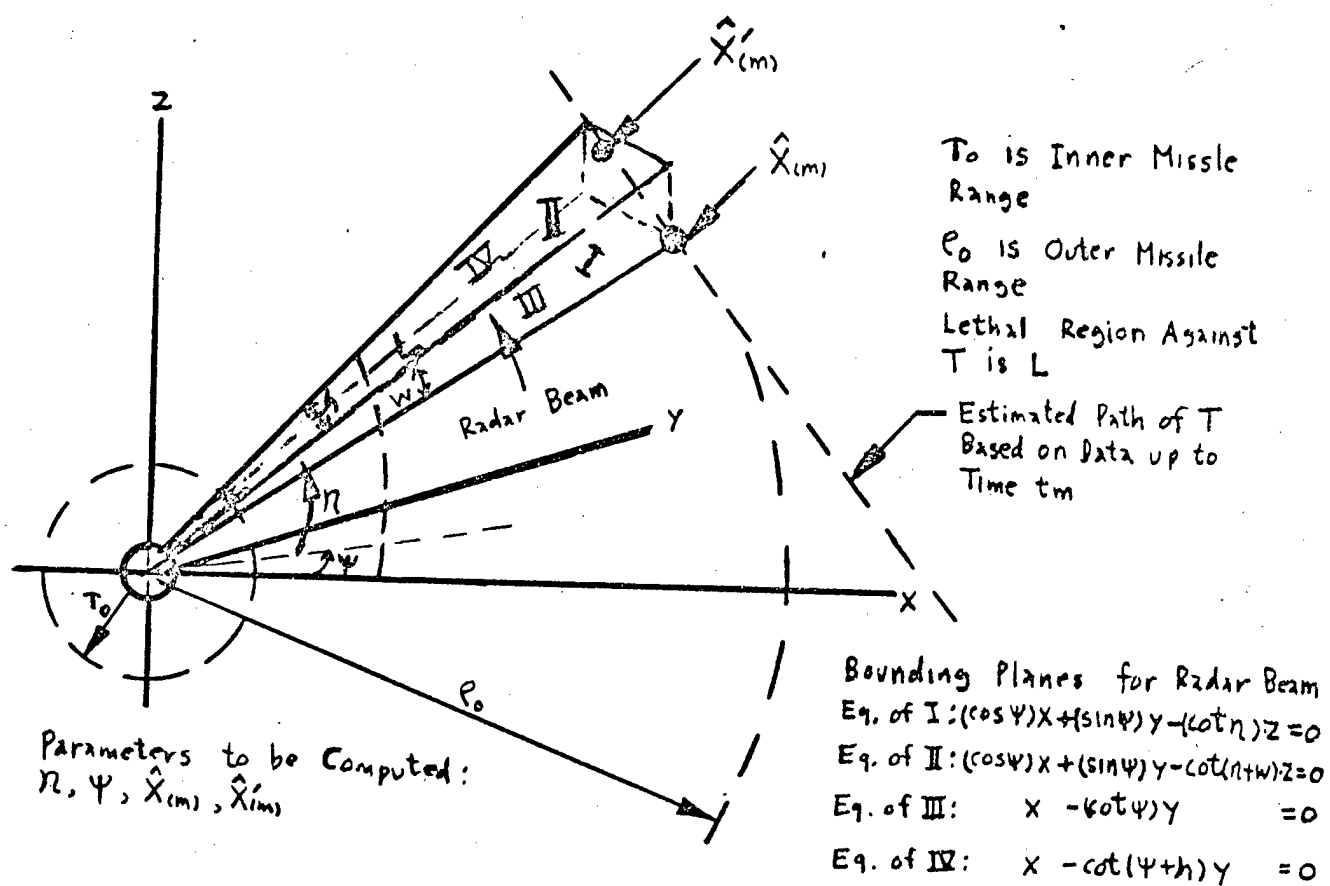$\eta, \psi, \hat{X}_{(m)}, \hat{X}'_{(m)}$

FIGURE 1.  LETHAL DETECTION REGION FOR FIRE CONTROL RADAR

# REFERENCES

(a) Naval Air Development Center SECRET Report No. NADC-SD-7078
"Expendable Jammer Effectiveness Analysis Presentation Report" (U)
of 10 Jan 1971

(b) Naval Air Development Center Confidential Report No. NADC-SD-7008
"Applications of Discrete Linear Regression Theory to a Comparison
of ASW Tracking Techniques" (U) of 27 Jan 1970

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## APPENDIX A

## OBTAINING OF OBSERVATIONAL DATA

In order to implement the model, the critical parameters, $\overset{o}{r}_i$, $\overset{o}{\theta}_i$, $\overset{o}{\theta}_i$ and $\sigma^2_{r_i}$, $\sigma_{\theta_i}$, $\sigma_{\phi_i}$, must be obtained. Figure A-1 illustrates the typical view of a radar operator (azimuth-range) when jamming is present.

Figure A-2 illustrates the relevant geometry when two independent radar sources simultaneously attempt to view the target with jamming present.

By elementary trigonometric considerations

$$\overset{o}{X}_i = \frac{a \cdot \sin \overset{o}{\theta}''_i \cdot \cos \overset{o}{\theta}'_i}{\sin (\overset{o}{\theta}''_i - \overset{o}{\theta}'_i)}$$

$$\overset{o}{Y}_i = \frac{a \sin \overset{o}{\theta}''_i \cdot \sin \overset{o}{\theta}'_i}{\sin (\overset{o}{\theta}''_i - \overset{o}{\theta}'_i)}$$

ground range $\overset{o}{r}_i = \sqrt{\overset{o}{X}_i{}^2 + \overset{o}{Y}_i{}^2} =$

$$\frac{a |\sin \overset{o}{\theta}''_i|}{|\sin (\overset{o}{\theta}''_i - \overset{o}{\theta}'_i)|} \sqrt{\cos^2 \overset{o}{\theta}'_i + \cos^2 \overset{o}{\theta}''_i}$$

For simplicity, employing a human observer, upper and lower bounds $\theta_i^{(2)}$ and $\theta_i^{(1)}$, respectively, are perceived so that a certain confidence of true $\theta_i$ within these bounds can be ascertained. For example, we might have $2\sigma_{\theta_i} \approx$ sample mean ( $\theta_i^{(2)}$ ) - sample mean ( $\theta_i^{(1)}$ ) for all i's and using Jacobians we can obtain $\sigma^2_{r_i}$ approximately:

$$\sigma^2_{r_i} \approx \left(\frac{\partial r_i}{\partial \theta'_i}\right)^2_{\left(\begin{array}{c}\theta'_i = \hat{\theta}'_i \\ \theta''_i = \hat{\theta}''_i\end{array}\right)} \cdot \sigma^2_{\theta'_i} + \left(\frac{\partial r_i}{\partial \theta''_i}\right)^2_{\left(\begin{array}{c}\theta'_i = \hat{\theta}'_i \\ \theta''_i = \hat{\theta}''_i\end{array}\right)} \cdot \sigma^2_{\theta''_i}$$

$$\text{Redundently,} \quad Cov \begin{pmatrix}\overset{o}{X}_i \\ \overset{o}{Y}_i\end{pmatrix} \equiv \begin{pmatrix}\sigma^2_{X_i} & \sigma_{X_i Y_i} \\ \sigma_{X_i Y_i} & \sigma^2_{Y_i}\end{pmatrix} \approx \frac{\partial(\overset{o}{X}_i, \overset{o}{Y}_i)}{\partial(\overset{o}{\theta}'_i, \overset{o}{\theta}''_i)} \cdot \begin{pmatrix}\sigma^2_{\theta'_i} & 0 \\ 0 & \sigma^2_{\theta''_i}\end{pmatrix} \cdot \left(\frac{\partial(\overset{o}{X}_i, \overset{o}{Y}_i)}{\partial(\overset{o}{\theta}'_i, \overset{o}{\theta}''_i)}\right)^T$$

etc.

(See page 4 for other relations between x and y coordinates.)
We need $\overset{\circ}{\rho}_i$, $\sigma^2_{\phi_i}$ and as alternate values, direct range $\overset{\circ}{\mathcal{R}}_i$, and $\sigma^2_{\mathcal{R}_i}$.
See figure A-3.

The extreme elevation heights of the target at time ti are

$$0 < h_i^{(1)} < h_i^{(2)}$$    , a priori known beforehand.  We let

$$\overset{\circ}{\mathcal{R}}_i \overset{df}{=} \tfrac{1}{2}(\mathcal{R}_i^{(1)} + \mathcal{R}_i^{(2)})$$

$$= \tfrac{1}{2}(\sqrt{\overset{\circ}{\rho}_i^2 + h_i^{(1)2}} + \sqrt{\overset{\circ}{r}_i^2 + h_i^{(2)2}})$$

and   $$\overset{\circ}{\phi}_i \overset{df}{=} \tfrac{1}{2}\left(\text{Arcsin}\left(\frac{h_i^{(1)}}{r_i^{(1)}}\right) + \text{Arcsin}\left(\frac{h_i^{(2)}}{r_i^{(2)}}\right)\right)$$

or, alternately,   $$\overset{\circ}{\phi}_i \overset{df}{=} \text{Arcsin}\left(\frac{\overset{\circ}{z}_i}{\overset{\circ}{\mathcal{R}}_i}\right)$$   , where

$$\overset{\circ}{z}_i \overset{df}{=} \overset{\circ}{h}_i \overset{df}{=} \tfrac{1}{2}(h_i^{(1)} + h_i^{(2)})$$

As before, we can obtain an estimate of   $\sigma^2_{\phi_i}$   by

$$2\,\sigma_{\phi_i} \approx \text{Arcsin}\left(\frac{h_i^{(2)}}{\mathcal{R}_i^{(2)}}\right) - \text{Arcsin}\left(\frac{h_i^{(1)}}{\mathcal{R}_i^{(1)}}\right)$$

Similarly,

$$2\,\sigma_{\mathcal{R}_i} \approx \mathcal{R}_i^{(2)} - \mathcal{R}_i^{(1)}$$   , etc.

We note that, in general, although when jamming is present, no
direct visual observation of the target can be made, the centered
coordinates of the jamming pattern; that is, the computed

$$\overset{\circ}{\theta}_i \,,\, (\overset{\circ}{\theta}_i{}' \,,\, \overset{\circ}{\theta}_i{}'') \,,\, \overset{\circ}{\phi}_i \,,\, \overset{\circ}{r}_i \,,\, \overset{\circ}{\mathcal{R}}_i \,,\, \overset{\circ}{x}_i \,,\, \overset{\circ}{y}_i \,,\, \overset{\circ}{z}_i$$

are taken as, in a sense, best uniform estimates of the target's
coordinates at time ti.  Hopefully, biasedness caused by this procedure
will be minimal.

A human factors study is needed to evaluate for each i, the upper
and lower bounds of the various parameters —

$$\overset{\circ}{\theta}_i^{(1)} , \overset{\circ}{\theta}_i^{(2)} \,(\, \overset{\circ}{\theta}_i^{(1)}{}' , \overset{\circ}{\theta}_i^{(2)}{}' , \overset{\circ}{\theta}_i^{(1)}{}'' , \overset{\circ}{\theta}_i^{(2)}{}'' \,) , h_i^{(1)} , h_i^{(2)}$$

and relate these meaningfully to confidence levels for the corresponding
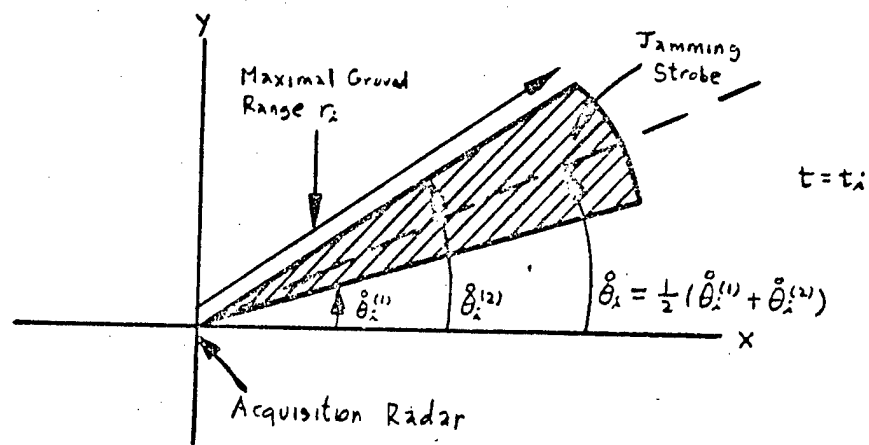true parameters unbiasedly.

FIGURE A-1.  AZIMUTH-RANGE VIEW OF RADAR OPERATOR
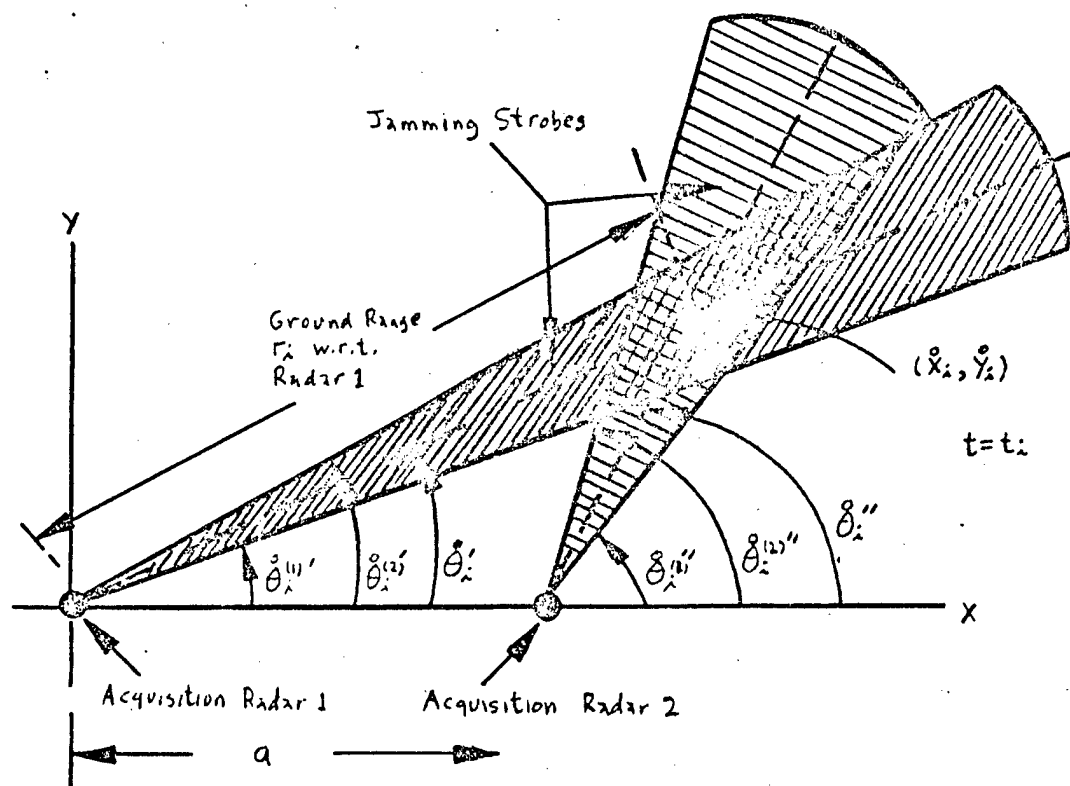WHEN JAMMING IS PRESENT

FIGURE A-2. GEOMETRY OF TWO INDEPENDENT AZIMUTH-RANGE VIEWS
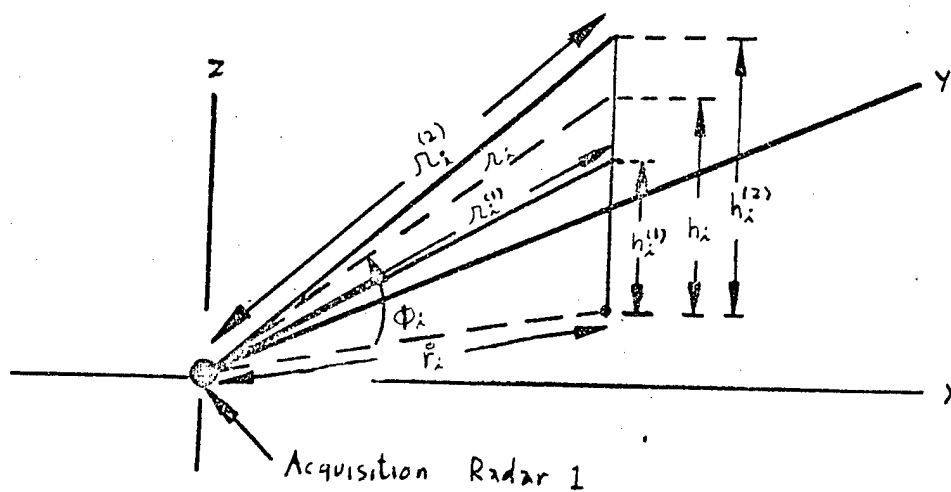BY RADAR OPERATORS WHEN JAMMING IS PRESENT.

FIGURE A-3.   ELEVATION PARAMETERS FOR ACQUISITION RADAR